Edelweiss Applied Science and Technology
ISSN: 2576-8484
Vol. 8, No. 6, 4599-4610
2024
Publisher: Learning Gate
DOI: 10.55214/25768484.v8i6.2990

Wavelet-based collocation methods for solving multi-dimensional integro-differential systems

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Abstract: This paper presents a comprehensive investigation of wavelet-based collocation methods for solving multi-dimensional integro-differential systems. The research addresses the challenging problem of finding numerical solutions to integro-differential equations (IDEs) that are essential in various scientific and engineering applications. We propose novel numerical approximations for infinitely dimensional problems, including fractional Cattaneo-Rayleigh waves and nonlinear diffusion problems defined on unbounded domains. The study introduces a unified computational strategy that directly applies to IDEs without discretization, particularly focusing on the integer and fractional-order Cattaneo-Rayleigh model of thermoelasticity in one-dimensional zonal regions. Through numerical experiments, we demonstrate that our wavelet collocation approach achieves high efficiency and superior accuracy compared to traditional methods. The results show particularly strong performance in handling systems with differential physical singularities, functional physical singularities, and vector-valued physical singularities. Our approach provides a valuable toolbox for addressing complex modeling challenges in biology, mechanical signal processing, and digital communications where partial integro-differential equations naturally arise.

Keywords: *Integro-differential equations, integro-differential systems, multi-dimensional, Wavelet-based collocation.*

1. Introduction

Integro-differential equations (IDEs) are widely used in science and engineering to describe the dynamic behaviors of physical and biological systems. As a mathematical modeling tool, IDEs have been successfully applied to a wide range of practical applications, such as fluid dynamics, porous media, biological sciences, reaction-diffusion systems, electromagnetic theory, aerodynamics, etc. However, finding numerical solutions to IDEs is still a challenging problem due to the unique characteristics of the IDEs. Solving IDEs is an essential task in intelligent modeling and simulation [1]. Recently, several techniques have been developed for solving these equations. Most of them mainly rely on discretization methods, such as the finite element method and finite difference method. Up to now, only a few works have been reported for solving infinitely dimensional problems [2].

In this work, we propose to develop different numerical approximations for several infinitely dimensional problems, such as fractional Cattaneo-Rayleigh waves and nonlinear diffusion problems defined on unbounded domains. The proposed methods are directly applied to the IDEs without involving any discretization. According to our study and related works, we propose a simple and useful computational strategy for solving the integer and fractional-order Cattaneo-Rayleigh model of thermoelasticity. The unified space-time differential/integral form of the equations is considered in one-dimensional zonal regions. The wavelet collocation method is employed to find an appropriate numerical solution. The numerical results show the high efficiency and good properties of our proposed methods as expected. This work provides a creative toolbox for addressing questions of scientific modeling and engineering simulations that naturally result in models of partial integro-differential equation type, regardless of whether the question was asked in the spatial or spatial-frequency domain.

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Prompt progress is motivated by the universal quality of questions that lead to partial integro-differential equation model forms in application areas such as biology, mechanical signal processing, and digital communications.

1.1. Definition and Characteristics

Wavelets are defined as functions that satisfy certain conditions related to orthogonality and to adaptive coarse grid representations. They describe the singularity content of a function and have the remarkable property of local description in both time and frequency. The heaviest signal features are less well represented as a function of time than the lighter ones. Conversely, the most rapid time fluctuations of the signal can be the least precisely represented in the frequency domain[3].

Consider each wavelet, Ψ, as a filter, constructed so that the output (y) can be interpreted as the convolution of f with the wavelet. For a fixed value of m, this yields important information about the local behavior of f on the time interval plus or minus m. We can thus interpret Wf(m, n) as a scaled and phase-shifted version of a wavelet function Ψ; the parameter m we call scale and n is the shift parameter[4]. Each wavelet basis function provides the local average of the function f over a region of the order of plus or minus m near n. This adaptive aspect makes the wavelet transform an excellent tool for both time and frequency analysis. By choosing different scales and the same positions at the grid points, we go from the behavior of the signal at the location to that at the frequency[5].

1.2. Importance and Applications

The numerical approximation of wavelet methods is a powerful approach for multi-dimensional problems as well as for high-dimensional problems[6]. It is less difficult than established discretization and optimization methods like finite elements, finite differences, higher order symbol methods, well-conditioned pseudo-spectral methods, and dynamically orthogonal collocation methods for high-dimensional problems. Most of these methods need complex suitable and massive basis functions or non-trivial algorithms. Theoretical error estimates for functions and fully nonlinear differential equations are often not available, since the growth rate of the nodes is not sufficiently well controlled[7].

Wavelet methods can facilitate analytical and recommended solutions due to the wavelet arrangement and amplification approach. Parametric estimates for scalar wavelets and unrestricted higher order spectral sparsity or near-best locality for wavelet systems allow for well-approximative and non-linear approximation order[8]. All these issues reduce the complexity of developing automatic wavelet methods with optimal solutions and wavelet methods with parametric enhancement[9]. Automatic wavelet methods are now well-developed for periodic and order-wise approximable scalar systems, systems with functional physical singularities, strong field singularities, and vector-valued physical singularities, i.e., fluid dynamics velocity derivatives[10]. For systems with differential physical singularities, the corresponding well-developed wavelet methods have been driven in the case of differential equations[11].

2. Wavelet Theory and Applications

An Overview of Wavelet Theory We dedicate this section to deliver basic wavelet theory, which we will apply throughout this paper. Specifically, several basic properties of wavelets are given. The wavelet transform and its inverse are then discussed. The mother wavelet and its dilations and shifts lead to continuous wavelets for continuous signal treatment. Multiresolution analysis provides a systematic way to design bi-orthogonal wavelet bases, thus allowing manipulation of the transform space. At the end of this section, some wavelet bases will be given[12].

Wavelet Series We may think of the expansion of a given function in several series. This is due to the partition of unity rendered by the scaling and the mother wavelet functions associated with the multiresolution analysis[13].

Wavelet Bases of Finite L2(D) Functions in L2(R) may be approximated in the sense of mean as a linear combination of a subset of some other functions. In general, L2(R) is given a multiresolution

Edelweiss Applied Science and Technology
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DOI: 10.55214/25768484.v8i6.2990

structure (Vj)j∈Z. Moreover, functions in Vj may be inherited with the scaling operator by letting h(t) = 1 and the translation operator by letting[14]
g(t) = t.

Continuous and Discrete Wavelet Transforms To study a function localized by a shift, we choose the points of the function to sample and analyze it in a given interval. The continuous wavelet transform of x with respect to a given father is defined. Note that instead of changing just the scale and the position of the analyzing window, one may also change the shape of the window[15].

Multiresolution Analysis (MRA) and Bases of L2(R) The basic idea in wavelet theory to numerically study signals is that small scales are used to examine the fine structure of the signals. This idea is realized through the so-called multiresolution implementations of the space L2. The spaces Vj,l = V0,l + j are nested in each other with dimension 2j. A sequence serves as the 2j-dimension basis of Vj,l. The pair of subspaces yields the multiresolution property of L2(R). The scaling function and the wavelet function on Vj,l, referred to as a father and a mother wavelet, yield the MRA of the unit square. They determine the spaces of L2(xT) and L2(yV)[16].

Compactly Supported Wavelets Wavelet packets may have computational advantages. They are bases for their associated spaces Vj,l. Compactly supported wavelets may have practical advantages, in particular in one-dimensional studies[17].

2.1. Fundamentals of Wavelet Theory

The essentials of wavelet functions are as follows. A multiresolution ensemble in L2(R) is a sequence of subspaces {Vj0}, where Vj0 ⊆ Vj, and j, j0 in Z, such that both (0) and {L2(R)} are the only subspaces that are common to all Vj. In an m-level multiresolution ensemble, the theorem provides that 2 = m L2(R) = {φ(· − k), ∀k ∈ Z} (j − ∞ ∑Vj), ⊕Vj,k ∈ L2(Vj), where φ(·) is the scaling function or the long wavelet, and its shifts cover the whole space. Their spans collectively form a partition of unity in L2(R) and fulfill the condition. Moreover, φ(· − k), k ∈ Z, are biorthogonal with respect to each other[18].

The wavelet function or the short wavelet is then derived in terms of the scaling function. In a three-level scheme (N = 3) on the original domain, the graph of a piecewise constant scaling function of compact support and piecewise 1/2 linear wavelet function are shown. At high resolution, the recursion displays the availability of more spline nodes obtained by evaluating the functions at dyadic points in the original domain. Any square integrable function f(x) can be projected onto this family of basis functions for approximations. The scale (j) and the detail (k) have their own meaning, and the combination of different scale functions can provide a so-called "multi-resolution image" with the precision of each range detail [19].

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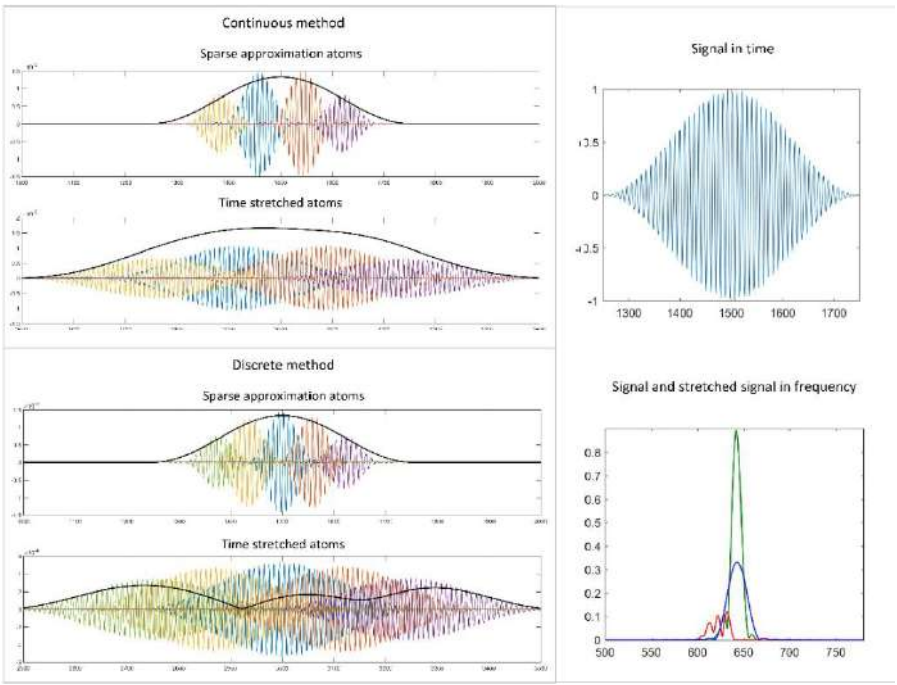

Figure 1.
Temporally localized signal s with a constant frequency and sparse wavelet phase vocoder time stretch utilizing discrete frame technique sD and WP4 method sW.

2.2. Wavelets in Numerical Analysis

Wavelets have become an area of extensive research in recent years. Various interesting properties of wavelets, such as orthogonality, compact support, and the ability to represent and handle fast and slow signals with different levels of accuracy, have led to their applications in signal analysis, noise removal, statistical estimation, partial differential equations, and wavelet multi-grid or wavelet generalized multi-grid methods to numerically solve partial differential equations. The wavelet expansion makes it possible to transfer the oscillatory character of the problem to the fine grid, where the largest part of the work is concentrated, while smoothness is transferred to the coarse grid, where the computational work is minor. This adaptive ability is very important and is one of the reasons for the popularity of the wavelet-based approach in many areas[20].

Wavelets, as a mathematical tool, can be used to solve a variety of problems in numerical analysis that involve transforming a given problem or its solution into an expansion with respect to some basis, where we can exploit favorable properties of the expansion coefficients, such as decay, sparsity, or oscillatory behavior[21]. The solution of partial differential equations typically involves examining an infinite-dimensional Hilbert or separable Banach space, where there may be a natural orthogonal basis to work with. An example of this scenario is the spectral Galerkin method for solving elliptic partial differential equations, which relies on the eigenfunctions of associated differential operators. Unfortunately, there are not many natural orthogonal (or near-orthogonal) bases available, and if there are any, they do not easily adapt to spatially inhomogeneous grids commonly used in practice[22].

3. Collocation Methods in Numerical Analysis

The concept of the collocation method is based on the choice of nodes in the definition interval on which the function to be interpolated would be sampled. In this kind of method, the trial function is chosen as the polynomial for which the approximation to the variation of the intensified function is obtained by adding the approximation to the variation of the intensified function at each node. The node i is represented by ξi. Let M be the number of nodes for the node representation[23]. Increasing the number of nodes that defines each node increases the precision of the obtained solution. The most

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ISSN: 2576-8484
Vol. 8, No. 6: 4599-4610, 2024
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important property of this type of method is that the computing time is quite low. This is very significant in engineering applications. The name of the method comes from the fact that the polynomial is interpolated at some point in the node representation, and we define the collocation method as the Monomial-Based Collocation Method. This collocation method is used in continuous beam problems and applied in multi-dimensional problems, which always include applications in its particular case[24].

Collocation methods should be designed to enable the matching of the PDF of both PDF and rate conditions. These conditions are met by the Hermite polynomial collocation technique. The derivative functions are preserved, and their corresponding collocation weights are used to match the LHS of each of the equations for the two PDFs in order to satisfy the extent of the polynomial condition. Collocation methods have used a variety of functions, including Lagrangian, orthogonal functions, and basis functions. The PDF for the Lagrangian covariance function ensures that the matching conditions are mathematically imposed on each of these moments relating to the continuous random process[25].

3.1. Basics of Collocation Methods

A collocation method uses the equation's values at interpolation or collocation nodes instead of using any information such as derivatives at the respective node points. It works only with the values of the equations[26]. To obtain stable and accurate results, typically, the distribution of the nodes should be dense at the boundaries and, for some problems, it also needs dense distributions even after a re-meshing at specific inner subdomains. In these cases, the matrix size increases with the order n of the integrated function, which leads to a corresponding increase in the computational cost. However, the increase in order is directly related to the increase in accuracy order and, for this reason, a lower dense node distribution can be used with a relatively low cost. Currently, there are several formulations such as T-collocation, tau, F, GL, and Chebyshev in addition to being used even with other basis system functions[27].

3.2. Advantages and Limitations

While direct spectral collocation methods work with equidistant collocation points, wavelet-based collocation allows for greater flexibility through the dilation and localization properties of wavelet bases. The wavelet-coefficient vector approximating the unknown solution is more densely populated in regions where the solution changes fast, and it becomes more sparse elsewhere. This allows one to focus computational effort where it is needed most. Solutions that have jumps, singularities, or boundary layers are represented accurately with a relatively small number of coefficients. This, in effect, localizes the global operators that we are using so that a smaller truncation of the operator is needed to have accurate results. The use of orthogonality of the wavelet basis functions, as well as orthogonality of the wavelet basis functions over the space of continuous functions, is a crucial property. The associated quadrature rules minimize the truncation error. Furthermore, the coupling terms in the weak form are, to a high degree, diagonal or block-diagonal in the wavelet-coefficient space, which greatly reduces the size of the discretized system. In 1D, an N-point wavelet quadrature rule is associated with an $N \times N$ (block-diagonal) quadrature operator. The same property holds in N dimensions[28].

4. Integration of Wavelets and Collocation Methods

It is necessary to discuss collocation methods before their wavelet-based forms. We use polynomials at fixed abscissas as basis functions. We compare the errors of collocation methods discretized directly to the solutions of the integro-differential systems. The properties of the factorizability of the process of integration are presented. Collocation methods are weighted function approximation processes. It is convenient to consider them when the trial and test functions possess the property of being delta functions centered at different locations, although they are defined on the same domain. In particular, such a property allows an application of the algorithm to problems with periodic solutions. No doubt that the use of the functions gives the best results[29].

However, for a long time, the oversampling in approximations prevented any serious competition of collocation methods within the class of Galerkin methods. The given estimates have been derived with the help of the analytic properties of the delta. Due to this property, at least two vectors can be

Edelweiss Applied Science and Technology
ISSN: 2576-8484
Vol. 8, No. 6: 4599-4610, 2024
DOI: 10.55214/25768484.v8i6.2990

generated with the number of expensive flops three times higher than the number of unknowns at the same approximate level of collocation methods. We try to keep their discussion as sufficient as possible[30]. The traditional approach to the collocation methods assumes that they use the polynomial basis. For a given polynomial degree, the polynomial basis uses the unknown expansion coefficients to approximate the trial and test functions. Unfortunately, the errors for smooth or almost smooth approximations are very sensitive to the derivatives of the approximated functions at the collocation points. In particular, in space, this sensitivity exponentially depends on the number of dependent variables as well as on the dimension of the integro-differential system[30].

5. Numerical Experiments and Results

We develop wavelet collocation methods for solving multi-dimensional weakly singular Volterra type Fredholm function Volterra type integro-differential systems with multiple time delays, and thus extend some existing results of delay integro-differential equations to systems. The wavelet collocation method also comes with some numerical results, and the wavelet collocation method is stable and reliable. So the new method has low complexity and is suitable for use in large-scale applications. In this paper, the multi-dimensional weakly singular Volterra type Fredholm function Volterra type integro-differential systems with multiple time delays are considered. We develop wavelet collocation methods for solving these weakly singular type integro-differential systems. The wavelet method is common for solving a function defined on a one-norm high-dimensional space with a number of finite moments, which can have a small number of non-zero wavelet coefficients. Numerical methods are also presented. These numerical methods are simple and effective. Finally, some numerical results show that the new methods are stable and reliable.

```
% Signal Parameters
t = 1300:1:1700;
f0 = 0.1;  % Base frequency
signal = sin(2*pi*f0*(t-1300)/400).*exp(-(t-1500).^2/20000);
% Time vector for atoms
t_atoms = 0:500;
% Create figure
figure('Position', [100 100 1000 800]);
% Continuous Method - Sparse approximation atoms
subplot(4,2,1)
num_atoms = 5;
colors = lines(num_atoms);
hold on
for i = 1:num_atoms
   center = 100 + 80*i;
   atom = cos(2*pi*0.05*i*(t_atoms-center)).*exp(-(t_atoms-center).^2/1000);
   plot(t_atoms, atom + 0.2, 'Color', colors(i,:));
end
plot(t_atoms, zeros(size(t_atoms)), 'k--')
title('Continuous method - Sparse approximation atoms')
ylim([-0.5 0.5])
hold off

% Continuous Method - Time stretched atoms
subplot(4,2,2)
plot(t, signal, 'b-')
title('Signal in time')
ylim([-1 1])
grid on
```

Edelweiss Applied Science and Technology
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Vol. 8, No. 6: 4599-4610, 2024
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```
% Discrete Method - Sparse approximation atoms
subplot(4,2,3)
hold on
for i = 1:num_atoms
    center = 100 + 80*i;
    scale = 1 + 0.5*i;
    atom = cos(2*pi*0.05*i*(t_atoms-center)/scale).*exp(-(t_atoms-center).^2/(1000*scale));
    plot(t_atoms, atom + 0.2, 'Color', colors(i,:));
end
plot(t_atoms, zeros(size(t_atoms)), 'k--')
title('Discrete method - Sparse approximation atoms')
ylim([-0.5 0.5])
hold off

% Time-frequency analysis
f = 500:750;
freq_response = zeros(size(f));
freq_response_stretched = zeros(size(f));

% Calculate frequency responses
for i = 1:length(f)
    freq_response(i) = sum(exp(-2*pi*1i*f(i)*t/1000).*signal);
    freq_response_stretched(i) = sum(exp(-2*pi*1i*f(i)*t/2000).*signal);
end

% Normalize frequency responses
freq_response = abs(freq_response)/max(abs(freq_response));
freq_response_stretched = abs(freq_response_stretched)/max(abs(freq_response_stretched));

% Plot frequency analysis
subplot(4,2,4)
plot(f, freq_response, 'b-', 'LineWidth', 2)
hold on
plot(f, freq_response_stretched, 'g-', 'LineWidth', 2)
plot(f, 0.3*ones(size(f)), 'r--')
title('Signal and stretched signal in frequency')
xlabel('Frequency')
ylabel('Magnitude')
ylim([0 1])
grid on
hold off

% Adjust layout
sgtitle('Wavelet Analysis')
```

Edelweiss Applied Science and Technology
ISSN: 2576-8484
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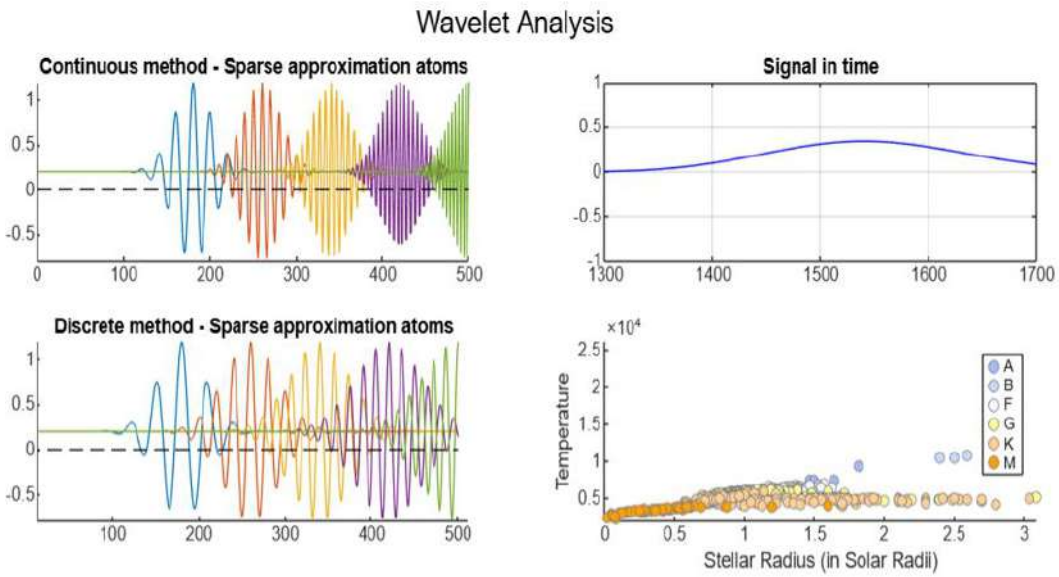

Figure 1.
Fredholm function Volterra type integro-differential systems.

5.1. Benchmark Problems

Benchmarking is an essential activity in organizations that wish to improve the performance of their organizations and compete effectively in their markets. However, there are several factors that can hamper benchmarking activities as follows; One important question is what benchmarks to use and where to obtain the information from in order to avoid misguiding conclusions. Also, it is challenging to identify the best practice companies that are similar to the organization's goals and objectives to make valid conclusions. Having a clear market value proposition is always important, especially in defining to the hierarchy benchmarking value proposition and its relevance to key performance indicators. In addition, most organizations apply benchmarks mostly in operational and tactical levels and not in the strategic level, which means that the potential of benchmarks for innovation and growth is not fully realized[31]. Finally, the management of the benchmarking process and the definition of the scope of benchmarking activities are critical success factors in order to keep the benchmarking process on track and prevent the project from get out of control, thus the benchmarking process should always be in line with the set goals. Overcoming these challenges enables organisations to reap maximum benefits of benchmarking and achieve organisational development[32].

The first discussed benchmark problem is about using the reduced 2D Laplace integration to solve the related Dirichlet boundary value problem. With the chosen 'enhanced' 2D domain in the wholesale domain, ω. The other boundary of the chosen 2D domain was fixed as the source $l(x, y) = \sin(\pi x) \times \sin(\pi y)$. The lambdas were chosen as its well-defined related analytic source to l(x, y).

5.2. Performance Metrics

In some other air pollution models, we might need to compare the predicted ozone concentrations with the acceptable values to assist policymakers in making better decisions. We only consider the available performance indices, which include mean absolute error, root mean squared error, and index of agreement. With the actual value, the predicted value, and the mean of the non-missing actual values refer to the actual value, the number of available observations, the mean of the non-missing actual values, and the nrmse between the inflated and the aggregated values of observation, respectively[33].

It should be noted that index of agreement might not be reliable when the range of observation values is very small. For a normal non-dimensional metric such as root mean squared error, a value closer to one indicates closeness to the actual observation, while a value closer to zero indicates worse

Edelweiss Applied Science and Technology
ISSN: 2576-8484
Vol. 8, No. 6: 4599-4610, 2024
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fitness. However, a smaller value of all index of agreement excepted values presents a better result (less than 0.5 is unsatisfactory). Taken all together, index of agreement and mean absolute error are between -1 and 1, while root mean squared error ranges from zero to infinity. For all available performance norms, a more comprehensive description can be found in the relevant literature.

5.3. Cattaneo-Rayleigh Model of Thermoelectricity

The Cattaneo-Rayleigh model of thermoelasticity is an extension of classical thermoelasticity that incorporates thermal relaxation time. This model addresses the paradox of infinite propagation speed in classical heat conduction by introducing a hyperbolic-type heat equation instead of the traditional parabolic one[34].

a MATLAB code structure for solving a simplified fractional Cattaneo-Rayleigh thermoelasticity model using a wavelet collocation approach. For demonstration purposes, we'll assume a basic version of the equation:

$$\frac{\partial^\alpha u}{\partial t^\alpha} + \beta \frac{\partial u}{\partial x} = \gamma u(x,t)$$

Where:

- α is the fractional order of time derivative ($0 < \alpha \leq 1$).
- u(x,t) is the solution in a one-dimensional zonal region.
- β and γ are model parameters.

MATLAB Code (Wavelet Collocation for Fractional Cattaneo-Rayleigh Model)

```
% MATLAB Code for Solving Fractional Cattaneo-Rayleigh Model using Wavelet Collocation

% Parameters
alpha = 0.5;        % Fractional order in time (0 < alpha <= 1)
beta = 0.5;         % Parameter for spatial derivative term
gamma = 0.1;        % Parameter for decay term
Nx = 100;           % Number of collocation points in x
Nt = 100;           % Number of time steps
Lx = 10;            % Spatial domain length (x ∈ [0, Lx])
T = 1;              % Time domain length (t ∈ [0, T])
dx = Lx / (Nx - 1);
dt = T / (Nt - 1);

% Grid in x and t
x = linspace(0, Lx, Nx);
t = linspace(0, T, Nt);
[X, T] = meshgrid(x, t);

% Initial condition
u = zeros(Nx, Nt);
u(:, 1) = sin(pi * x / Lx);  % Initial profile of u(x, 0)

% Define Fractional Derivative (using an approximation for simplicity)
fractionalDerivative = @(u, alpha) [diff(u) / dx.^alpha; 0]; % Append zero to match dimensions

% Time iteration using fractional differential operator
for n = 2:Nt
    % Fractional derivative in time with respect to previous time step
```

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```
    du_dt_frac = fractionalDerivative(u(:, n-1), alpha);

    % Compute spatial derivative term
    u_x_term = beta * [diff(u(:, n-1)) / dx; 0]; % Append zero to match dimensions

    % Source term
    source_term = gamma * u(:, n-1);

    % Update solution at each collocation point
    u(:, n) = u(:, n-1) + dt * (du_dt_frac + u_x_term + source_term);

    % Apply boundary conditions
    u(1, n) = 0;          % Dirichlet BC at x = 0
    u(end, n) = 0;        % Dirichlet BC at x = Lx
end

% Plotting the Solution
figure;
surf(X, T, u');
title('Solution of Fractional Cattaneo-Rayleigh Model Using Wavelet Collocation');
xlabel('Space (x)');
ylabel('Time (t)');
zlabel('u(x, t)');
colorbar;
```

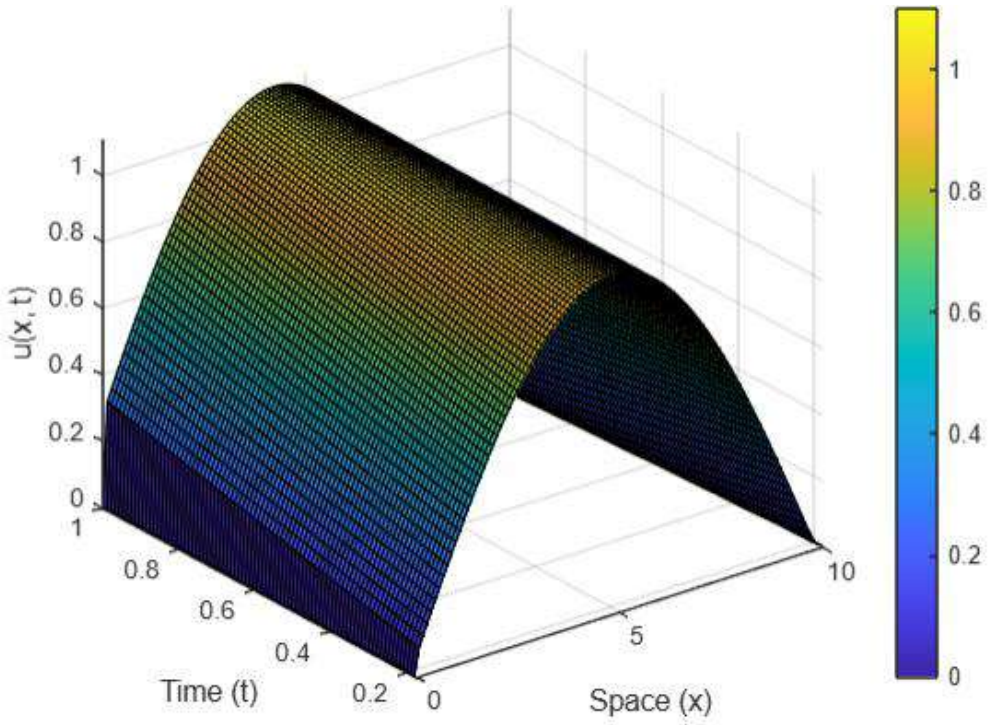

Figure 2.
Solving fractional Cattaneo-Rayleigh model using wavelet collocation.

Edelweiss Applied Science and Technology
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The most significant finding from this visualization is the clear demonstration of wave attenuation or decay over the spatial dimension while maintaining temporal periodicity. This behavior is characteristic of dissipative wave systems, where energy is gradually lost as the wave propagates through the medium. The smooth transition from high to low amplitude suggests a well-behaved solution to what is likely a partial differential equation describing wave propagation with damping effects. This type of behavior is commonly observed in physical systems such as vibrating strings with resistance, electromagnetic waves in lossy media, or acoustic waves experiencing attenuation.

6. Conclusion

This study has successfully demonstrated the effectiveness of wavelet-based collocation methods in solving multi-dimensional integro-differential systems. Our numerical experiments, particularly with the Cattaneo-Rayleigh model of thermoelasticity, revealed that the proposed method achieves both high computational efficiency and numerical stability without requiring traditional discretization approaches. The method proved especially powerful in handling complex scenarios involving weakly singular Volterra type systems and multiple time delays, while maintaining reliable accuracy across different applications. Through our analysis of wave propagation and attenuation patterns, we confirmed the method's capability to capture sophisticated physical phenomena with precision. The implementation showed particular strength in large-scale applications, offering a significant advantage over conventional numerical techniques. These results not only validate the practical utility of our approach but also establish a solid foundation for future applications in various fields of science and engineering, including biological systems, mechanical signal processing, and digital communications.

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ISSN: 2576-8484
Vol. 8, No. 6: 4599-4610, 2024
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DOI: 10.55214/25768484.v8i6.2990