

## Characterization of the mineral fracture mechanics to improve the efficiency in the grinding process in a concentrator plant

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**Abstract:** This study addresses the critical need for improving energy efficiency in the mineral comminution process, a key factor in the mining industry. Despite the extensive research on comminution, significant gaps remain in understanding how energy consumption can be optimized across different stages of the process. The objective of this research is to evaluate energy consumption patterns and identify strategies for enhancing energy efficiency in mineral processing, particularly focusing on ball mills. The methodology involved an analysis of operational data from a processing plant, including energy consumption rates, feed characteristics, and processing parameters. Key findings reveal that energy consumption in comminution is influenced by factors such as ore hardness, mill parameters, and the size reduction achieved. Furthermore, the study indicates a notable potential for reducing energy use by optimizing milling conditions and applying energy-efficient technologies. In conclusion, this research provides valuable insights for reducing energy consumption in the mining sector, with implications for sustainability, cost reduction, and environmental impact. Future studies should explore alternative technologies and further refine operational models to enhance energy efficiency in mining operations.

**Keywords:** Ball mills, Energy consumption, Energy efficiency, Mineral comminution, Sustainable mining.

### 1. Introduction

The fragmentation of rocks and minerals is a critical aspect of the mining industry, particularly in the processes of comminution, such as crushing and grinding, which are essential for liberating valuable minerals from the gangue. In this context, the application of fracture mechanics emerges as an essential tool for understanding material behavior under stress, especially during ore size reduction. Fracture mechanics provides theoretical principles that explain crack initiation and propagation, allowing engineers to predict material behavior during comminution [1, 2]. The application of these principles is key to optimizing energy consumption in ore processing plants, particularly concentrator plants, where energy demands are high. A more precise understanding of mineral fracture properties is a challenge that contributes to the sustainable development of mining operations.

Globally, the mining industry faces increasing pressure to reduce energy consumption and environmental impact, especially considering that grinding alone can account for up to 50% of total energy used in mineral processing [3, 4]. Several studies have explored the application of fracture mechanics to predict particle behavior during comminution, such as those by Austin and Klimpel [5]. Despite advances in these areas, many mining operations still rely on generalized models that overlook the specific fracture properties of materials, leading to inefficient energy use and increased production costs [6].

In Peru, the problem is more pronounced due to the mineralogical diversity of the deposits and the lack of technological adaptation in many concentrator plants. According to the Ministry of Energy and Mines in Perú, energy consumption in comminution processes in Peru represents over 60% of the total energy cost in some medium-scale mining operations. However, applying fracture mechanics to optimize these processes remains limited. Studies highlight the absence of mineral-specific characterization protocols, which hinders the implementation of tailored strategies.

This research aims to bridge the gap between the theoretical concepts of fracture mechanics and their practical application in grinding processes in Peruvian concentrator plants. The focus is on identifying mechanical variables that influence the breakage of specific minerals and utilizing them to improve process efficiency and reduce energy consumption. The research question guiding this study is: How can the characterization of fracture mechanics in minerals contribute to increasing energy efficiency in grinding processes in concentrator plants in Peru? Addressing this question will provide valuable insights for the national mining sector and contribute to more sustainable and cost-effective operations.

## 2. Theoretical Framework

### 2.1. Conceptual and Theoretical Definitions

Fracture mechanics is a branch of solid mechanics that studies the formation, propagation, and behavior of cracks in materials subjected to mechanical stress. In mining engineering, this discipline is essential for understanding how different types of rocks respond to crushing and grinding processes, which are intended to reduce particle size and liberate valuable minerals. One of the most commonly used theoretical models in this context is the Bond Work Index, which evaluates the energy required to reduce a material to a specific size based on its hardness and resistance to fracture [3]. Other relevant concepts for the study include toughness, elastic modulus, and hardness—material properties that determine how easily a mineral break under stress [7, 8]. These parameters help classify materials according to their fragmentation behavior and are essential for designing efficient comminution circuits.

Several empirical studies have shown clear correlations between the mechanical properties of ores and the energy efficiency of the grinding process. For example, Kick [9] found that ores with higher fracture toughness tend to consume more energy during size reduction. In Latin America, grinding processes based on mineralogical behavior led to a 15% reduction in energy consumption in a medium-scale Peruvian concentrator plant. These data support the argument that characterizing fracture properties is not only theoretically relevant but also offers measurable operational benefits.

### 2.2. General and Specific Theoretical Approaches

Three key theoretical approaches are particularly relevant to this study:

Griffith's Theory of Fracture Griffith [1] which postulates that fracture occurs when the energy released by crack propagation exceeds the energy required to create new surfaces. This theory laid the groundwork for modern fracture mechanics and is fundamental to understanding breakage in brittle materials such as rocks.

Bond [3] Comminution Theory Bond [3] which provides an empirical relationship between energy input and particle size reduction. Though simplified, it remains widely used in the mining industry to estimate energy requirements based on ore hardness.

Gonzalez [10] highlights that contemporary copper mining in Chile—and globally—is facing new challenges, particularly in seeking higher processing rates and resource efficiency due to the declining ore grades in existing deposits. In this context, thermal pretreatment prior to the grinding stage emerges as a promising strategy, as it can significantly enhance comminution operations by reducing energy requirements and improving overall processing efficiency.

Schönert [11] demonstrated that, although cement is typically regarded as a brittle material, particles of just a few micrometers in size can undergo plastic deformation without fracturing when subjected to stress. This observation challenges traditional assumptions about material behavior at

micro scales and suggests that particle size plays a critical role in determining the mechanical response of brittle materials.

The Population Balance Model (PBM) Austin [12] which offers a statistical and predictive model for breakage kinetics during grinding. This model helps simulate how particles of different sizes behave during milling and how fracture properties affect final particle size distributions.

Each of these theories provides a specific perspective on the fracture process, from energy-based models to predictive simulation techniques. Together, they create a solid theoretical foundation for evaluating how mineral properties influence grinding performance.

### *2.3. Synthesis and Theoretical Justification*

In summary, the theoretical framework of this study integrates key definitions of fracture mechanics with empirical findings that demonstrate the impact of mineral properties on energy consumption in grinding. The selected theories—Griffith's fracture mechanics, Bond's energy model, and the Population Balance Model—offer complementary perspectives that support the relevance of characterizing the mechanical behavior of ores. The literature reveals a consistent relationship between fracture resistance and comminution efficiency, both at the laboratory and industrial levels. However, a theoretical and practical gap persists in adapting these principles to specific regional contexts, such as Peru, where geological variability demands customized processing strategies. This research contributes to filling that gap by offering a framework for incorporating fracture mechanics into the design and operation of more energy-efficient concentrator plants.

## **3. Methodology**

### *3.1. Research Design and Approach*

This study adopts a bibliographic qualitative approach with an exploratory and documentary design Hernández, et al. [13] aimed at synthesizing existing scientific knowledge related to the fracture mechanics of minerals and its application in improving grinding efficiency in concentrator plants. The qualitative nature of the research lies in the interpretive analysis of theoretical, technical, and empirical documents, while the exploratory component seeks to identify and understand variables and factors not yet extensively studied within the Peruvian mining context. The documentary design involves reviewing and analyzing books, scientific articles, theses, technical reports, and institutional publications that address the mechanical behavior of rocks and the efficiency of comminution processes.

### *3.2. Data Collection Techniques*

The primary technique for data collection was documentary analysis. Scientific databases such as Scopus, ScienceDirect, SpringerLink, and Google Scholar were systematically explored, using keywords such as fracture mechanics, mineral grinding, comminution energy, rock breakage, and Peruvian mining. The inclusion criteria for the documents included: (a) relevance to the central topic of the study, (b) publication in indexed journals or reputable scientific platforms, and (c) publication dates between 2000 and 2024, although seminal works prior to that date (e.g., Bond, Griffith) were also considered due to their foundational value. The selected documents were organized using a reference manager (Zotero), allowing for efficient classification and thematic coding.

### *3.3. Information Analysis Procedure*

The analysis of the selected information was carried out through qualitative content analysis, focusing on the identification of categories and recurring themes. Specifically, four analytical axes were defined:

- (a) conceptual definitions of fracture mechanics,
- (b) mechanical properties of minerals relevant to grinding,
- (c) empirical correlations between fracture properties and energy consumption, and
- (d) documented cases of application in mining processes.

A comparative matrix was used to cross-reference the findings, highlighting convergences, contradictions, and theoretical gaps. Additionally, statistical summaries from previous empirical studies were extracted to complement the qualitative analysis with relevant quantitative data (e.g., energy consumption percentages, reduction in operational costs).

### 3.4. Scope and Limitations of the Study

Due to biosafety restrictions, no *in-situ* or laboratory experiments were conducted; therefore, the research is based exclusively on secondary data. While this limits the possibility of direct generalization to specific operational contexts, the strength of the study lies in the integration and critical analysis of high-quality international and national sources. Furthermore, the research offers a theoretical and methodological foundation for future empirical studies focused on the Peruvian mining sector, especially in regions where energy efficiency in mineral processing remains a pending challenge.

## 4. Results

In Peru, mining operations primarily process polymetallic ores composed of mineralogical species containing metals such as lead, zinc, copper, gold, silver, among others. Energy consumption during mineral processing is primarily attributed to electric power usage, which is essential for heating processes and other operational needs. As illustrated in Figure 1, the highest energy consumption occurs in the grinding stage (45%), followed by crushing (10%). Thus, comminution (crushing and grinding) accounts for more than 55% of the total energy consumption in a concentrator plant. Regarding energy consumption within the comminution process, it is distributed as follows:

- Elastic deformation of the particle,
- Plastic deformation of the particle prior to fracture,
- Friction between particles.

In mining activities, electric power is used as an energy source for mineral processing operations and for transporting ores from the mines to the beneficiation areas. On the other hand, according to Table 1, it can be observed that energy consumption is directly related to the size of the processing plant—that is, larger plants tend to consume less energy per ton of processed mineral.

**Table 1.**  
Energy consumption in mineral concentrator plants by size.

Classification	Plant Capacity (Tons per Day)	Specific Energy Consumption (kWh/ton)
Small-scale plants	50 to 100	2.5 to 3.5
Medium-scale plants	600 to 5,000	1.5 to 2.5
Large-scale plants	More than 5,000	1.2 to 1.8

Source: FONAM, Energy Diagnostics in SÍLICES INDUSTRIALES COMERCIALES S.A., 2015.

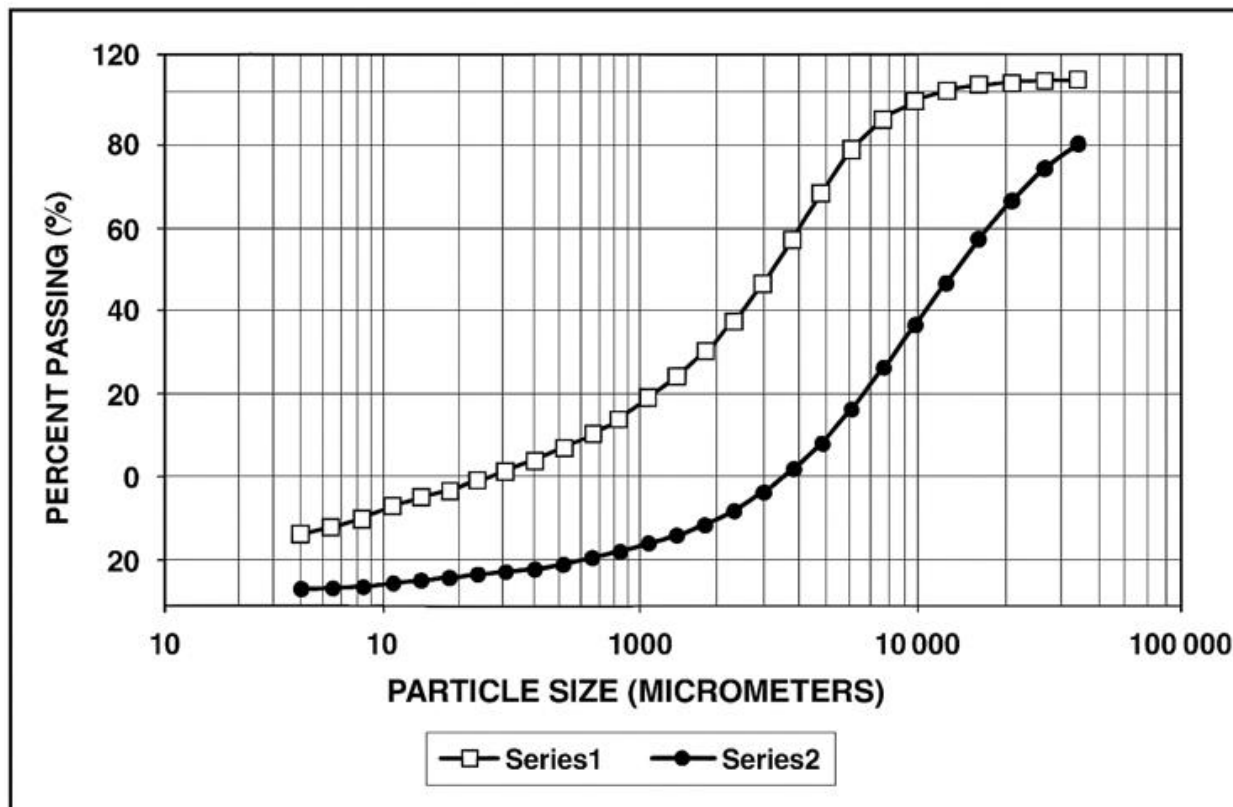
**Table 2.**  
Energy consumption in the crushing process.

Process Stage	Soft Rock (kWh/Ton)	Hard Rock (kWh/Ton)
Primary crushing	0.3 to 0.6	0.7 to 1.2
Secondary crushing	0.4 to 0.8	0.9 to 2.0

Source: Mining Manual – ESTUDIOS MINEROS DEL PERÚ SAC.

As shown in Table 2, as the size of the mineral decreases, electrical energy consumption increases.

The ore extracted from the mine presents various particle sizes, ranging from less than 1 millimeter (mm) to fragments larger than 1 meter in diameter. Therefore, the goal of the crushing process is to reduce larger fragments to a uniform maximum size of  $\frac{1}{2}$  inch (1.27 centimeters). The extracted material passes through the primary, secondary, and tertiary crushers, where it reaches a size smaller than  $\frac{1}{2}$  inch.



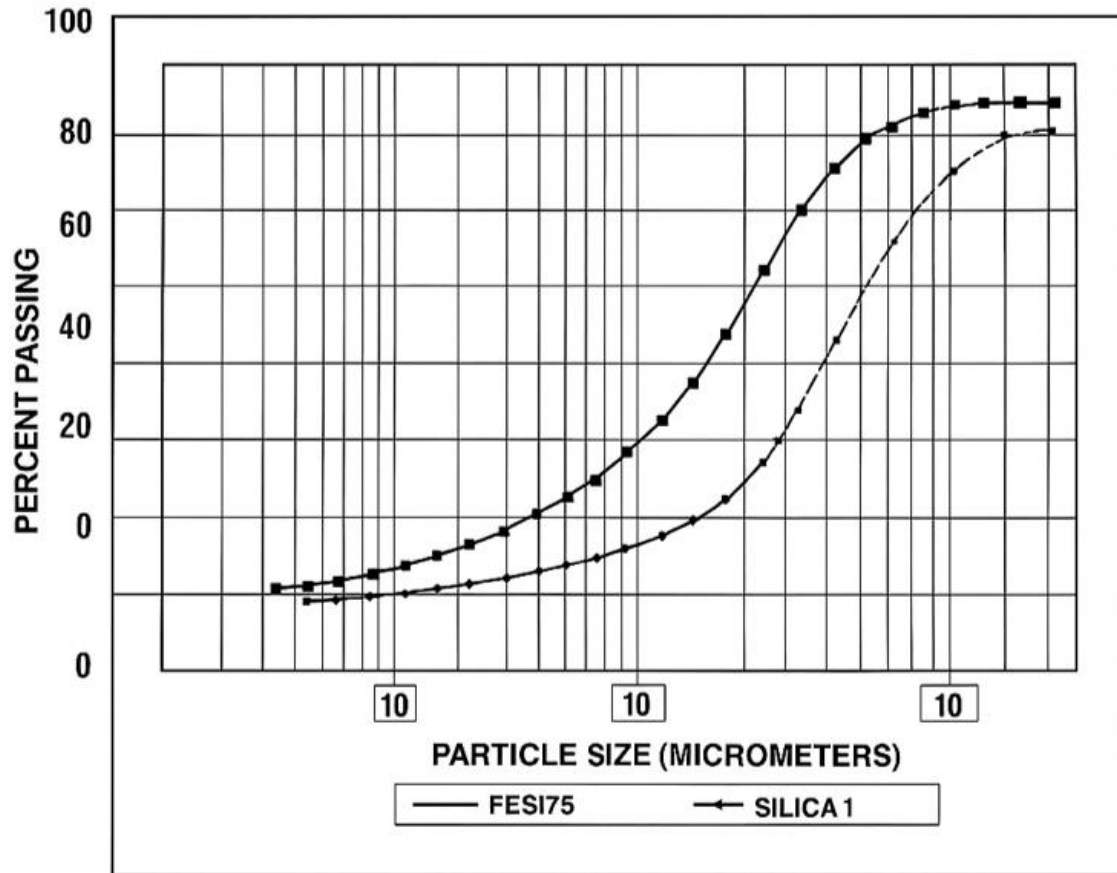
**Figure 1.**  
Particle size distribution of polymetallic ore (feed and product) from the gyratory crusher (8'x11').

**Table 3.**

Operating data of the gyratory crusher (8'x11'), used to calculate the equipment's energy consumption and the efficiency of energy use in reducing the feed ore size.

Parameter	Value
Feed rate (TMS/h)	13.00
F80	141,652.00
P80	15,679.00
Motor power (Hp)	24.00
Motor voltage (V)	440.00
Power factor (cos $\phi$ )	0.78
Nominal motor current (A)	30.00
Energy consumption (kWh/TM)	0.84
Reduction ratio	9.0
Total energy consumption (kWh)	10.90
Total energy consumption (Hp)	14.62
Work index (kWh/TMS)	15.74
Maximum throughput per day (tons/day)	512.39
Electric motor efficiency (%)	60.92
Practical current (A)	18.34

Experimental grinding tests and granulometric analysis curves of polymetallic ore are performed as a standard procedure within the mineral comminution process, with the aim of determining the degree of liberation of the sulfide mineral.



**Figure 2.**  
Particle size distribution of polymetallic ore (feed and product) from Ball Mill No. 2 (8'x10').

**Table 4.**

Operating data of Ball Mill No. 2 (8'x10'), used to calculate the energy consumption of the equipment and the efficiency of energy use in reducing the feed ore size.

Parameter	Value
Critical speed	38.32
Diameter (feet)	4.00
Energy consumption (kWh/TM)	9.19
Length (feet)	4.00
Reduction ratio	44.13
Moisture content (%)	6.59
Total energy consumption (kWh)	15.32
Feed rate (TMS/h)	1.67
Total energy consumption (Hp)	20.54
Motor power (Hp)	46.00
Work index (kWh/TMS)	20.95
Motor voltage (V)	440.00
Maximum throughput per day (tons/day)	89.63
Power factor ( $\cos \phi$ )	0.67
Electric motor efficiency (%)	44.66
Nominal motor current (A)	67.00
Operating hours	24
Ball charge level (%)	45
F80	16,550
P80	375
Practical current (A)	30.00

Experimental grinding tests and particle size analysis of polymetallic ore were conducted to determine the F80 and P80 of sulfide minerals, a crucial step for optimizing the grinding process and understanding mineral liberation.

**Table 5.**

National electric energy production showing the energy resources used (GWh) in Peru, generated by various sources. December 2019 and 2020.

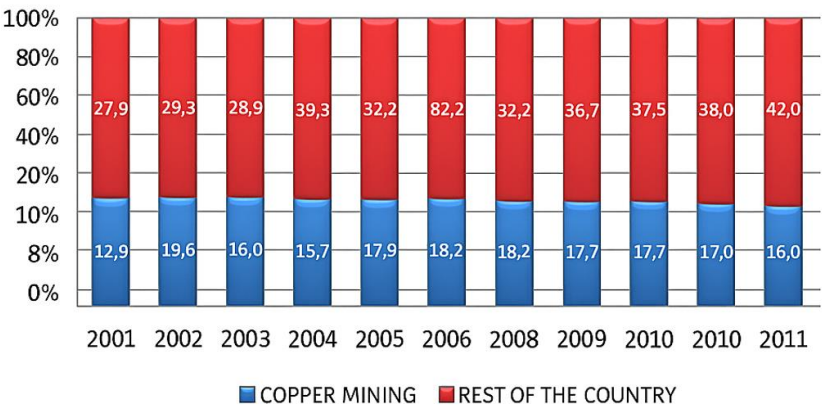
Energy Source	December 2019	December 2020	Variation (%)
Hydropower	3,262	2,845	-13%
Natural Gas	1,319	1,724	31%
Diesel/Coal/Residual	118	58	-51%
Bagasse/Biogas	47	51	9%
Wind	132	147	12%
Solar	79	74	-6%
Steam (Cogeneration)	0.35	0.26	-26%
Total National	4,957	4,899	-1.2%

Table 5 presents the indicators of electricity generation according to the type of energy resource used. In this context, hydroelectric units generated 2,845 GWh, representing a 13% decrease compared to the same month in 2019. On the other hand, thermal units powered by natural gas produced 1,724 GWh, which corresponds to a 31% increase compared to December of the previous year.

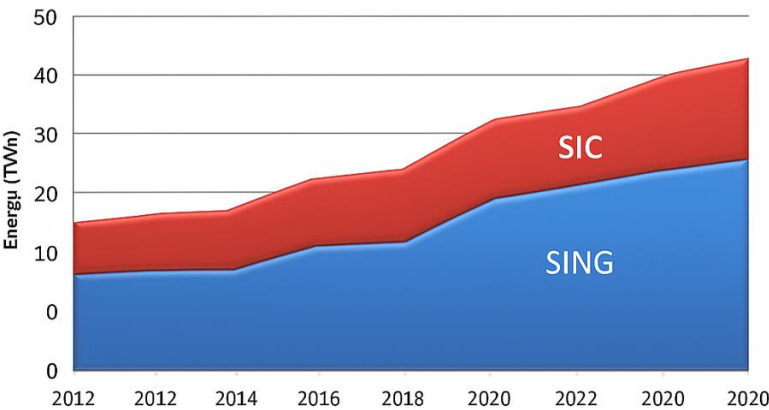
In comminution processes, energy efficiency is low [14] and concentrator plant design and operation are generally carried out at suboptimal energy efficiency levels [15]. Key factors contributing to this low energy efficiency in existing mineral processing operations include the variability in ore hardness and particle size, which leads to process inefficiencies. The output of the refined product is directly related to the available power. The energy consumption KPI is described in terms of [kWh/ton

of processed material], while material throughput is described as t/h (tons per hour) [14]. The average energy consumption for a mine is approximately 6,700 kWh/kton [14].

In terms of projected national maximum energy consumption, it was estimated that by 2020 the total demand would reach 51.7 TWh, with 33.9 TWh attributed to the SING (Sistema Interconectado del Norte Grande) and 17.7 TWh to the SIC (Sistema Interconectado Central). Concentrator plants would remain the main electricity consumers, accounting for 61% of the total energy demand. Additionally, desalination plants and pumping systems would gain importance, reaching 12% of electricity consumption by 2020.



**Figure 3.**  
National electricity consumption in the mining industry over time.  
**Source:** Prepared by COCHILCO.



**Figure 4.**  
Projected national electricity consumption in the mining industry over time.  
**Source:** CNE and Minist of energy.

Electric energy is essential for mining activities, where the use of electrical power helps minimize the specific energy consumption, that is, the amount of electricity used per unit of mineral product obtained—and consequently, the energy costs associated with this consumption. Improvements in efficiency can be achieved with minimal fluid energy usage, allowing companies to meet their productivity and competitiveness goals, while also enhancing profitability and reducing operational costs.



## 5. Discussion

One of the primary challenges in the mining industry is improving energy efficiency during mineral comminution processes, particularly in crushing and grinding stages. According to Bond [3] and Bond [16] comminution requires a substantial amount of energy, and inefficiencies often arise due to inadequate equipment sizing or incorrect operational parameters. The theoretical frameworks developed by pioneers such as Kick [9]; Von Rittinger [17] and Charles [18] emphasize the correlation between energy consumption and particle size reduction, laying the foundation for understanding the behavior of different ore types during size reduction operations. These classical theories have been refined over time to evaluate grindability more precisely, as demonstrated in Berry and Bruce [19] work and the practical testing methodologies proposed by Deister [20] and Rowland [4].

Subsequent advancements in grinding theory, such as those introduced by Austin and Klimpel [5] and elaborated further by Austin [12] underscore the importance of particle size distribution in mill design and simulation. Their research aligns with the findings of Menéndez Aguado [21] who applied mathematical modeling to assess energy use in fragmentation. Similarly, studies by Coello-Velázquez and Tijonov [22] as well as by Coello Velázquez [23] emphasize the need for regularity and optimization in mineral milling, especially when dealing with lateritic ores. Complementary works by Prasher [7] and Cáceres [24] provide practical approaches to improving energy efficiency through operational adjustments and process design.

Further empirical investigations conducted in Latin America, particularly in Peru and Chile, provide localized insights into energy consumption patterns in grinding circuits. Research by Álvarez-Rodríguez [25]; Castillo [26] and Zumaran [27] demonstrates how simulation tools and critical variables such as mill speed, fill level, and particle size distribution affect energy demand. Bernedo [28] also contribute valuable data through their case studies on ball mill control systems and process optimization. Meanwhile, Sepúlveda and Gutiérrez [29] along with Valenzuela Piñeiro [30] emphasize the significance of energy audits and efficiency strategies in Chilean mining operations, offering concrete proposals for reducing electricity consumption through engineering-based interventions.

Finally, modern perspectives on mineral processing reflect a more holistic understanding of the interaction between ore properties and process dynamics. Studies like those of Porras [31] and Echavaudis and Pérez [32] integrate the rheology of mineral suspensions and optimal particle sizing to enhance flotation outcomes and energy savings. Authors such as Maurice and Kenneth [8] and Fuerstenau and Han [33] provide foundational texts that compile decades of theoretical and experimental knowledge, essential for designing more sustainable mineral processing systems. This comprehensive body of literature not only reinforces the necessity for energy-efficient technologies but also highlights the role of rigorous experimental analysis and simulation in achieving meaningful improvements in operational performance.

## 6. Conclusions

This research underscores the critical role of energy consumption in the mineral processing sector, particularly within the context of crushing and grinding operations. The findings confirm that these processes represent the highest energy consumers within concentrator plants, with grinding alone accounting for 45% of the total energy consumption. The energy demand in these stages is closely tied to the characteristics of minerals, such as hardness and size, which significantly influence energy efficiency. This highlights the need for a more comprehensive understanding of energy dynamics and the implementation of more energy-efficient technologies to optimize mineral processing operations.

The results also show that larger concentrator plants, while processing more material, tend to be more energy-efficient per ton of ore processed. However, this study also reveals the challenges posed by mineral variability, particularly in relation to harder ores, which require more energy for size reduction. This reinforces the importance of adopting energy-efficient practices and advanced technologies tailored to specific mineral characteristics and plant capacities. Additionally, the growing energy demand in the mining sector, coupled with increasing national energy consumption trends, suggests that future

improvements in energy efficiency will be crucial for reducing environmental impacts and operational costs.

Looking ahead, future studies should focus on exploring innovative solutions to reduce energy consumption in mineral processing, especially in the context of renewable energy integration. While this research highlights the importance of optimizing energy use in mining operations, it also points to the need for more detailed investigations into the long-term impacts of energy-efficient practices and their economic viability. A limitation of this study is its reliance on available data from specific plants, which may not fully represent the diversity of mineral types and plant configurations across the industry. Nevertheless, these findings provide a solid foundation for further research aimed at improving energy management practices in the mining industry.

### Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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